

# CBCS SCHEME

USN 

A	V	P	9	O	E	C	O	8	Q
---	---	---	---	---	---	---	---	---	---

18MAT41

Fourth Semester B.E. Degree Examination, July/August 2022

## Complex Analysis, Probability and Statistical Methods

Time: 3 hrs.

Max. Marks: 100

*Note: Answer any FIVE full questions, choosing ONE full question from each module.*

### Module-1

- 1 a. Derive Cauchy-Riemann equation in Polar form. (06 Marks)
- b. Find the analytic function  $f(z)$  whose real part is  $x \sin x \cosh y - y \cos x \sinh y$  (07 Marks)
- c. If  $f(z)$  is analytic show that  $\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] |f(z)|^2 = 4 |f'(z)|^2$  (07 Marks)

OR

- 2 a. Find the analytic function  $f(z)$  given that the sum of its real and imaginary part is  $x^3 - y^3 + 3xy(x - y)$  (06 Marks)
- b. Find the analytic function  $f(z) = u + iv$  if  $v = r^2 \cos 2\theta - r \cos \theta + 2$  (07 Marks)
- c. If  $f(z)$  is analytic function then show that  $\left\{ \frac{\partial}{\partial x} |f(z)| \right\}^2 + \left\{ \frac{\partial}{\partial y} |f(z)| \right\}^2 = |f'(z)|^2$  (07 Marks)

### Module-2

- 3 a. State and prove Cauchy's Integral formula. (06 Marks)
- b. Evaluate  $\int_0^{2+i} \bar{z}^2 dz$  along (i) the line  $y = \frac{x}{2}$  (ii) The real axis to 2 and then vertically to  $2 + i$ . (07 Marks)
- c. Find the bilinear transformation which maps the points 1,  $i$ ,  $-1$  onto the points  $i$ ,  $0$ ,  $-i$  respectively. (07 Marks)

OR

- 4 a. Discuss the transformation  $w = e^z$ , with respect to straight lines parallel to  $x$  and  $y$  axis. (06 Marks)
- b. Using Cauchy's integral formula evaluate  $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ , where  $c: |z| = 3$  (07 Marks)
- c. Find the bilinear transformation which maps the points  $0, 1, \infty$  on to the points  $-5, -1, 3$  respectively. (07 Marks)

### Module-3

- 5 a. A random variable  $X$  has the following probability function for various values of  $X$ .

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k <sup>2</sup>	2k <sup>2</sup>	7k <sup>2</sup> +k

Find i) k    ii)  $P(X < 6)$     iii)  $P(3 < X \leq 6)$  (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- b. Out of 800 families with 5 children each, how many families would you expect to have  
 (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys (iv) atmost 2 girls, assuming equal probabilities for boys and girls. (07 Marks)
- c. The length in time (minutes) that a certain lady speaks on a telephone is a random variable with probability density function

$$f(x) = \begin{cases} Ae^{-x/5} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find the value of the constant A. What is the probability that she will speak over the phone for (i) More than 10 minutes (ii) Less than 5 minutes (iii) Between 5 and 10 minutes. (07 Marks)

OR

- 6 a. Find the constant C such that the function  
 $f(x) = \begin{cases} Cx^2, & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$  is a probability density function. Also compute  $P(1 < x < 2)$ ,  $P(x \leq 1)$  and  $P(x > 1)$  (06 Marks)
- b. 2% fuses manufactured by a firm are found to be defective. Find the probability that the box containing 200 fuses contains  
 (i) No defective fuses (ii) 3 or more defective fuses (iii) At least one defective fuse. (07 Marks)
- c. If  $x$  is a normal variate with mean 30 and standard deviation 5 find the probabilities that  
 (i)  $26 \leq x \leq 40$  (ii)  $x \geq 45$  (iii)  $|x - 30| > 5$   
 Given that  $\phi(1) = 0.3413$ ,  $\phi(0.8) = 0.2881$ ,  $\phi(2) = 0.4772$ ,  $\phi(3) = 0.4987$  (07 Marks)

**Module-4**

- 7 a. The following table gives the ages (in years) of 10 married couples. Calculate Karl Pearson's coefficient of correlation between their ages:

Age of husband (x)	23	27	28	29	30	31	33	35	36	39
Age of wife (y)	18	22	23	24	25	26	28	29	30	32

- b. In a partially destroyed laboratory record of correlation data only the following results are available:  
 Variance of  $x$  is 9 and regression lines are  $8x - 10y + 66 = 0$ ,  $40x - 18y = 214$ . Find  
 (i) Mean value of  $x$  and  $y$   
 (ii) Standard deviation of  $y$   
 (iii) Coefficient of correlation between  $x$  and  $y$ . (07 Marks)
- c. Fit a parabola of the form  $y = ax^2 + bx + c$  for the data (07 Marks)

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

(07 Marks)

OR

- 8 a. Obtain the lines of regression and hence find the coefficient of correlation of the data:

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

- b. Show that if  $\theta$  is the angle between the lines of regression (06 Marks)

$$\tan \theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \left( \frac{1-r^2}{r} \right)$$

(07 Marks)

- c. Fit a straight line  $y = a + bx$  to the data

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

(07 Marks)

**Module-5**

- 9 a. The joint probability distribution of the random variables X and Y is given below.

	Y	-4	2	7
X				
	1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
	5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Find (i)  $E[X]$  and  $E[Y]$  (ii)  $E[XY]$  (iii)  $\text{cov}(X, Y)$  (iv)  $\rho(X, Y)$ .

Also, show that X and Y are not independent.

(06 Marks)

- b. A manufacturer claimed that atleast 95% of the equipment which he supplied to a factory confirmed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 of them were faulty. Test his claim at a significance level of 1% and 5% ( $z_{0.05} = 1.96$ ,  $z_{0.01} = 2.58$ ). (07 Marks)
- c. A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure ( $t_{0.05}$  for 11 d.f. is 2.201) (07 Marks)

**OR**

- 10 a. Define the terms :

(i) Null hypothesis (ii) Type-I and Type-II errors (iii) Significance level (06 Marks)

- b. In an experiment of pea breeding the following frequencies of seeds were obtained:

Round Yellow	Wrinkled Yellow	Round Green	Wrinkled Green	Total
315	101	108	32	556

Theory predicts that the frequencies should be in proportions 9:3:3:1

Is the experiment in agreement with theory ( $\chi^2_{0.5}$  for 3 d.f is 7.815)

(07 Marks)

- c. The joint probability distribution of two discrete random variable X and Y is given by  $f(x, y) = k(2x + y)$  where x and y are integers such that  $0 \leq x \leq 2$ ,  $0 \leq y \leq 3$ . Find k and the marginal probability distribution of X and Y. Show that the random variables X and Y are dependent. Also, find  $P(X \geq 1, Y \leq 2)$ . (07 Marks)

\*\*\*\*\*